Homework Section 4.0 - Due 8th March

1. Suppose $f(x)$ is a periodic function with $p = 8$ and $f(1) = 4$, $f(0) = -6$, $f(-2) = -2$. If possible, compute

   (a) $f(-20)$.
   (b) $f(56)$.

2. Suppose $f(x)$ is a periodic function with $p = 7$ and $f(85) = 13$, $f(214) = 12$, $f(35119) = 11$. If possible, compute

   (a) $f(0)$.
   (b) $f(1)$.
   (c) $f(2)$.

3. Sketch the graph of

$$f(x) = \begin{cases} 2 & \text{if} \quad x \in [2k, 2k+1) \\ -3 & \text{if} \quad x \in [2k-1, 2k) \end{cases} \qquad \text{for all } k \in \mathbb{Z}$$

   over the interval $[-5, 5)$.
   Is this function periodic? If so, what is its period?

4. The amplitude of a periodic function $f(x)$ is given by $a = \frac{|\max(f(x)) - \min(f(x))|}{2}$. Calculate the amplitude of

   - $f(x)$ as in question 3.
   - $f(x) = \sin(x)$.
   - $f(x) = \cos(x)$.

   Is it possible to have a function with amplitude of -1? Answer by either giving an example or explaining why it is impossible.

5. Mark the following true or false, explain your answer briefly in sentences:

   (a) x_1 and x_2 are coterminal angles, then $\sin(x_1) = \sin(x_2)$.
   (b) If $\sin(x) = \sin(x')$, then $x = x'$.
   (c) The period of $\cos(x)$ is $[0, 2\pi)$.
   (d) The period of $\sin(x)$ is $2\pi, 4\pi, 6\pi, \ldots$
   (e) If $f(x)$ is a periodic function with $f(x) = 0$ for every integer x, then the period of $f(x)$ must be 1.
   (f) If t and s have the same reference angle, then $\sin(t) = \sin(s)$.
   (g) $f(x) = \sin(x^2)$ is an even function.

6. Give an example of an odd function and an even function that are *not* trigonometric functions.

7. Functions are not integers, they don't have to be even or odd.
   Give an explicit example of such function. (Hint: Lines)

8. Functions are not integers, they can be both even and odd.
   Give an explicit example of such function. (Hint: Don't think too hard.)